FUNICULAR STRUCTURES USING TOPOLOGICAL ASSEMBLIES

URVI SHETH[1] and AYSHA FIDA[2]
[1,2]*CEPT University*
[1]*urvi.sheth@cept.ac.in* [2]*ayshafida.p@gmail.com*

Abstract. Presented work is inspired by the research on funicular structures by Block Research Group and customising bricks by the first author. The research is focused on developing a mortarless construction system for funicular structures using topological assemblies on site. To make the proposed system financially viable in the India, it is suggested to limit the customisation of the topological modules. Topological assemblies interlock with its contact surfaces (Tessman, 2012). Further these force locked elements are kinematically constrained using an extrados post tensioning. As a result, the system is stable not only in complete compression, but it can also withstand lateral loads and vertical upliftment. Additionally, it is quick to assemble and dismantle the structure without foundation and by using minimum scaffolding. Therefore, the construction system can be used to build a range of temporary as well as permanent structures like temporary exhibition halls, emergency shelters, earthquake resistant structures, etc.

Keywords. Funicular structures; Mortarless masonry; Topological assembly; Interlocking modules; Limited customisation.

1. Introduction

Advancements in computational design and digital fabrication techniques opened immense possibilities of generating forms through structural concepts, integration of material behaviour and construction of funicular vaults. Several studies are conducted on form finding of funicular structures of both types, symetrical and asymetrical. In the last decade researches by organisations like Block Research Group has developed interactive digital tools which establish the relation between force and form in compression curved surface structures. These tools offer an intuitive and playful interface (Rippman M, 2016). Researches using these tools have demonstrated examples of designed and built projects.

RhinoVault plugin for Rhinoceros 5 is one such plugin that works on the principle of Thrust network Analysis (TNA) for funicular form generation (Rippman M, 2016). The generated forms are being constructed using a variety of techniques and materials. For example, traditional Catalan vaulting with thin brick shells, advanced stereotomic design with stone, ceramic tile construction, etc. (Rippman, Block 2013). Presented work uses RhinoVAULT to generate forms and investigate the construction possibility using topological assemblies

RE: Anthropocene, Proceedings of the 25th International Conference of the Association for Computer-Aided Architectural Design Research in Asia (CAADRIA) 2020, Paper 72 (Preprint).

2. Topological Assemblies

Topological assemblies are a type of interlocking system with solid and repetitive blocks which are kinematically constrained. The assemblies interlock with its contact surfaces. The planer topological assemblies were developed by the Institute of Material Science, Technical University Clausthal, Germany Tessmann 2012). These solids interlock three dimensionally, purely based on their geometric shapes. These assemblies are made of repetitive polyhedrons (Fig 1a) and osteomorphic blocks (Fig 1b). There are two types of topological interlocking, platonic solids and osteomorphic blocks. The first is interlocked with planar surfaces while the second has non-planar interlocking faces. Osteomorphic blocks interlock due to their matching concavo-convex surfaces. Given fixed boundary conditions the assemblies are able to resist high bending forces and even tension without any additional binding material like mortar (Fig 1c) (Dyskin et al. 2003).

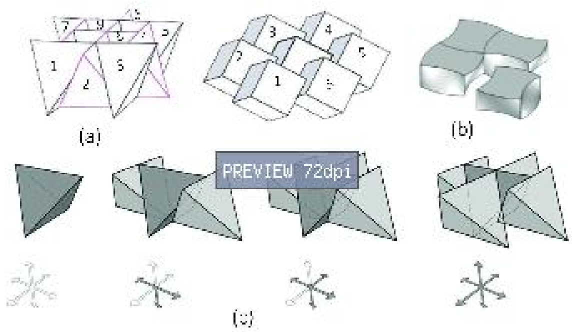

Figure 1. (a) Assembly of repetitive polyhedrons (Belenov - kernel et al. 2009), (b) Assembly of Osteomorphic blocks (Weizmann et al. 2011). (c) Tetrahedron configuration with interlocking properties (Tessmann 2012).

3. Module Design

3.1. FORM

Based on the osteomorphic topological assembly, a module is proposed. The module design is based on a Cosine/Sine trigonometric identity that resembles an osteomorphic block by D A Robinson. The proposed module has following curve character,

$$z(x, y) = 0.5h\cos\left(\frac{x}{B}\right)\sin\left(\frac{y}{B}\right), where B \leq x < 3B, 0 \leq y < 2B, h = B \quad (1)$$

The proportions of the module is maintained as B x B x B/2, where B is a variable value that determines the height of the block (Fig 2b).

The geometry of module is symmetrical along the central axis (Fig 2c, 2d). Thus, when the module is rotated by 180 degrees, the non-planar surfaces of the rotated module matches with the original module (Fig 2e) which ensures the interlocking of modules in two dimensional plane. These modules are defined as "standard module" here.

Figure 2. (a) Osteomorphic block by D A Robinson, (b) Proposed module, (c) Curves u and u' that defines the geometry (d)Top and bottom surface of the geometry. (e) Interlocking of modules (f) Linear assembly of modules. (g) Module assembly with angle spacers to achieve a curvature. (h) Types of modules (i) Assembly of modules along a catenary arch.

Unlike planer assembly (Fig 2f), to build curved geometry like centenary arch, funicular barrel vaults and symmetrical funicular forms, the system either demands 100% customisation or a limited customisation. A sequence of alternate standard module and "angle spacer" is proposed for the curved geometry (Fig 2g). The wedge shaped modules corresponding to the range of angles between two standard modules are called "angle spacers". The size and geometry of angle spacer varies with reference to the span, rise and curvature (single or double curvature) of the forms. Among the angle spacers, "Base module" and "Key stone" are named specifically (Fig 2h). These four types of modules make an interlocking system resulting in the elimination of mortar as well as limiting customisation of modules for construction (Fig 2i).

3.2. SIZE

To fix the size of the proposed form, a structural simulation was done using Scan-and-Solve plugin to Rhinoceros 5.0. The plugin allows structural simulation of bonded assemblies consisting of any collection of solid geometry using Finite Element Method (FEM). Total displacement, principle compression and tension were calculated for catenary arches using different module sizes. Following parameters are considered to run the simulation:

1. As generated in RhinoVAULT, structure is in complete compression.
2. Dynamic parameter 1: Size of module (Module 1: 10cm x 10cm x 10cm; Module 2: 15cm x 15cm x 15 cm; Module 3: 20cm x 20cm x 20cm)
3. Dynamic parameter 2: Span and Rise (Case 1: x1=x2= x; Case 2: x1=x2= 2x; Case 3: x1= x2= 4x; where x1 is span, x2 is rise and x= 75cm)
4. Material: M20 concrete

Simulation showed similar behaviour for total displacement in all three cases and it was negligible for all the cases (Fig 3a). Therefore, it is not considered while determining the size. Based on the simulation results of principle compression and tension (Fig 3b), maximum span and height up to 150cm was achieved safe with Module 3.

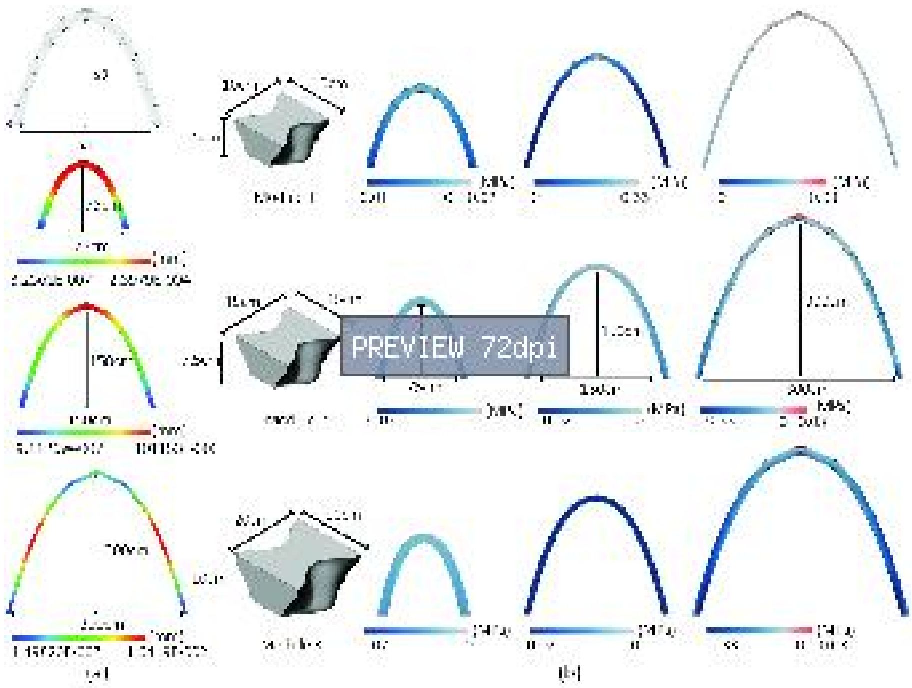

Figure 3. (a) Total Displacement in arch with respect to Case 1, Case 2 and Case 3 (b) Principle Compression and Tension in Catenary arch for Case 1, 2 and 3 using Module 1, 2 and 3.

Module 3 was further optimised for its thickness from 20cm to 10cm.

4. Test Cases

4.1. CATENARY ARCH & FUNICULAR BARREL VAULT

Catenary arches of span 300cm are simulated using module size 20cm x 20cm x 10cm to calculate total displacement and principle compression and tension (Fig 4). Rise of these arches are defined as, Case 1a (x1= 4x, x2 = x); Case 1b (x1=4x, x2 =2x); Case 1c (x1= 4x, x2 = 3x), where x1 is span, x2 is rise and x= 75cm.

Figure 4. (a) Total displacement (b) Principle Compression and Tension in Catenary arch.

These catenary arches were repeated horizontally for 5 and 10 times to create funicular barrel vaults. Principle compression and tension of the same are shown below (Fig 5a to 5b)

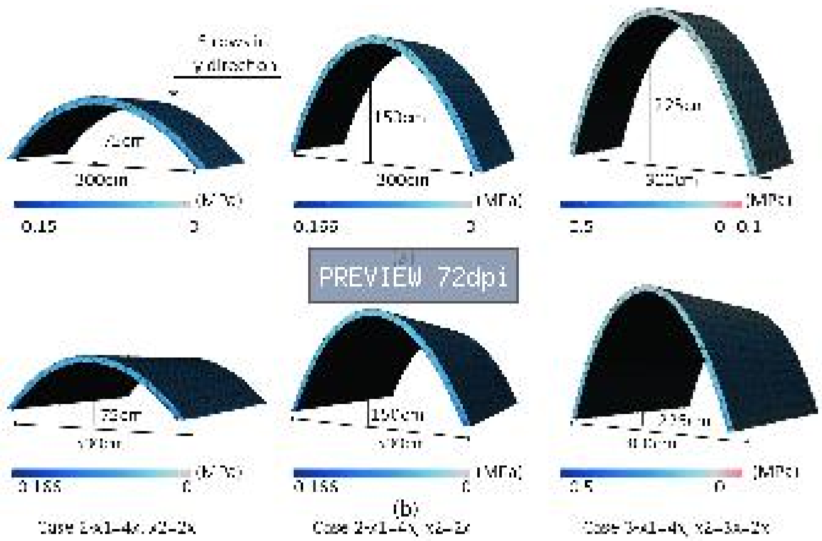

Figure 5. Principle compression and tension (a) Barrel Vault with 5 rows of catenary arches (b) Barrel Vault with 10 rows of catenary arches.

Inference: Height of the catenary arch should be less than 3/4 of the span. Barrel vault is in complete compression for the span 300cms and maximum rise 150cm. Tension zone is observed at the top of the vault for 225cm rise. Hence the rise of the structure should be equal to or less than 1/2 of the span. The behavior of vault is independent of the number of horizontal repetation.

4.2. FUNICULAR FORM WITH TWO PARALLEL SUPPORTS

Funicular form is generated with two parallel supports measuring 180cms. The module size is 20cm x 20cm x 10cm and span 300cm. Rise varies as, Case 2a (x1= 4x, x2 = x); Case 2b (x1=4x, x2 =2x), where x1 is span, x2 is rise and x= 75cm. Two alternative assembly methods, one where modules are assembled vertically and the other with horizontal assembly, were tested for total displacement (Fig 6) and principle compression and tension (Fig 7).

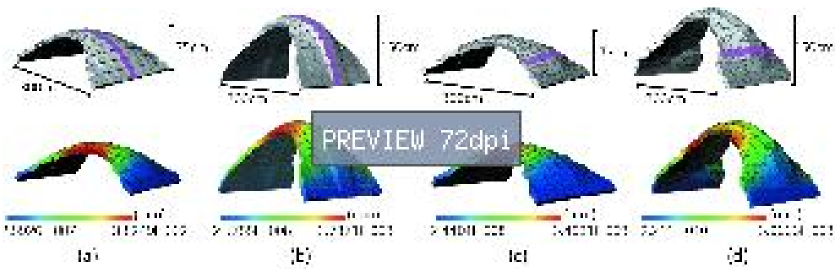

Figure 6. Total displacement (a, b) Modules assembled vertically (c, d) Modules assembled horizontally.

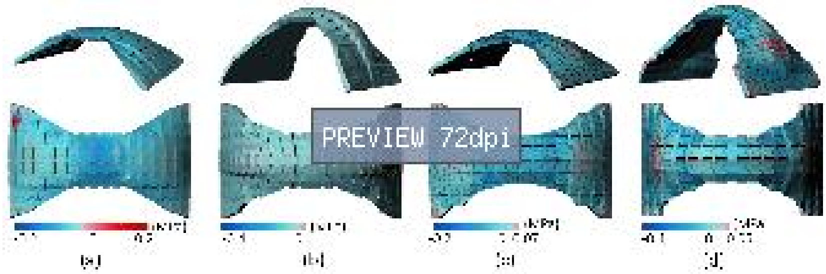

Figure 7. Principle compression and tension in funicular form with two parallel supports (a, b) Modules assembled vertically (c, d) Modules assembled horizontally.

Inference: For the span of 300cm, structure is in complete compression using horizontal assembly for the rise of 75cm and using vertical assembly for the rise of 150mm. Therefore, for such forms, assembly logic would vary case wise.

4.3. FUNICULAR FORM WITH FOUR LINEAR SUPPORTS

Funicular form is generated with four linear supports, each measuring 300cms. The module size, span and varying rise same as the previous case. Figure 8 show principle compression and tension.

Figure 8. Principle compression and tension in funicular form with four linear supports .

Inference: Vault is in complete compression for the span of 300cm and rise of 75cm. For the rise of 150cm, the entire assembly had several gaps between modules which led to tension in the structure. Therefore, further simulations were done by filling these gaps for a range of span 300cm to 450cm and rise 75cm to 150cm (Fig 10)

Figure 9. (a) Detail of filling the gaps between standard modules.

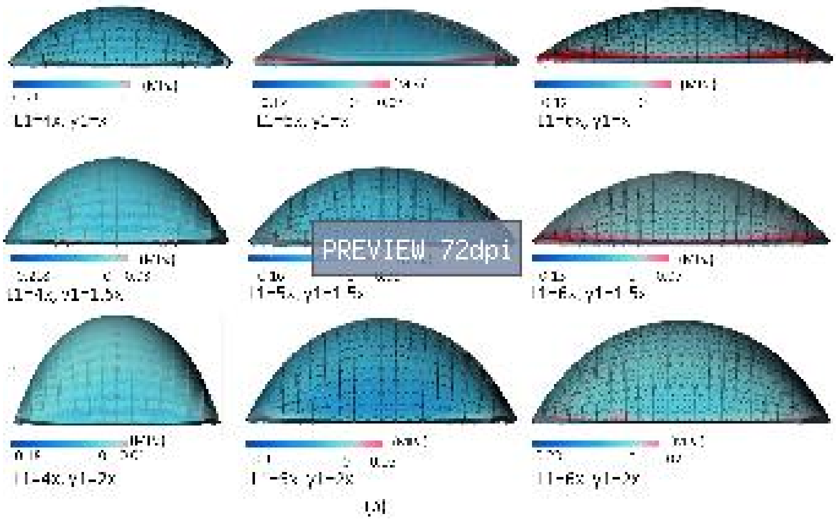

Figure 10. Principle compression and tension in funicular form with four side supports.

By filling the gaps, the assembly is bonded and the performance of the structure improved. It is in complete compression for the maximum span of 450cm and rise 150cm. Filler material is yet to be decided.

5. Construction logic

5.1. BARREL VAULT

Two methods of construction are visualised through scale 1:10 prototypes for Funicular barrel vault. Method 1, without scaffolding and Method 2, with limited scaffolding. In both the cases, modules were assembled by staggering standard modules and angle spacers alternatively. The staggering of block ensures that the modules are mutually locked with each other and construction joints are not continuous. It also eliminates the need of a continuous scaffolding. The entire assembly is kinematically constrained by extrados post tensioning.

Construction method 1 (Fig 11a to 11d): A catenary arch is drawn on ground along which modules were assembled horizontally. Once the first arch is completed, it is kinematically constrained with the tension cable fixed at its base before the next layers are assembled. The process is repeated for successive layers. On completion of the required number of layers, entire assembly is turned vertical and placed in its position with the help of a crane or a mechanical system.

Construction method 2 (Fig 11e to 11h): Scaffolding is required to assemble the first catenary arch in its position on site. Once the first arch is assembled, it is kinematically constrained with the tension cable fixed at its base before the next layers are assembled. The scaffolding can now be removed and the next rows of arches can be built without scaffolding. Staggering of modules create a pocket for the next block and ensures interlocking. The process is repeated for the required number of rows for the vault.

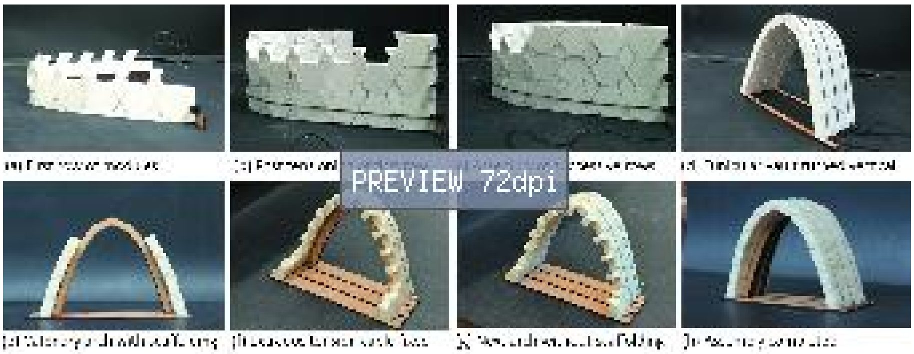

Figure 11. Construction method 1 and 2 for barrel vault.

5.2. FUNICULAR FORM WITH FOUR LINEAR SUPPORTS

For such symmetrical form having a square plan, the assembly begins from the outer edge having two parallel supports for the first arch. In case the base plan was rectilinear with similar support condition, the assembly would follow the line diagram of the trust network to ensure force flow through the structure. Similar to construction method 2 of barrel vault, modules are staggered and the first arch requires scaffolding. The rest can be assembled without continuous scaffolding. Supports can be provided at strategic locations. Refer Fig 12

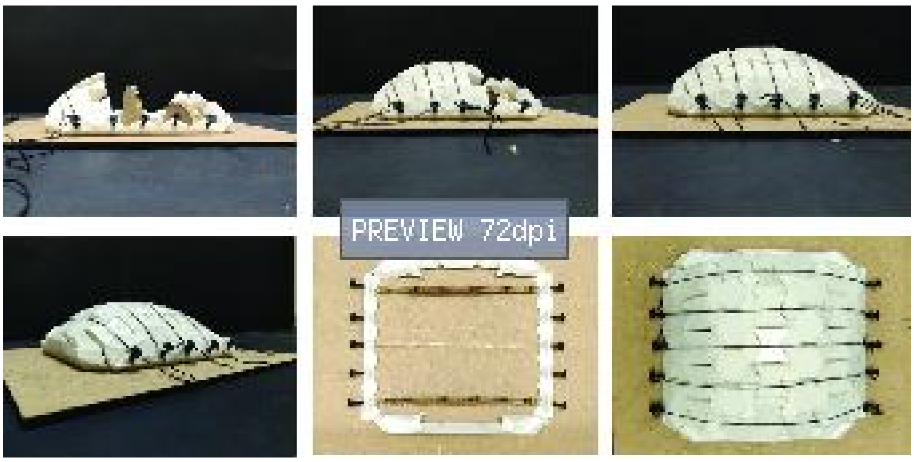

Figure 12. Construction method for funicular form with four sides support.

6. Conclusion

The construction system developed is **self-supporting, dry construction for symmetrical funicular forms using topological assemblies**. These are force locked elements which are kinematically constrained using an extrados post tensioning. Because of this, the system is stable not only in complete compression but **it can also withstand lateral loads and vertical upliftment**. The modules can be assembled and dismantled as required and transported to various locations. Therefore, the construction system can be used to build a range of temporary as well as permanent structures like temporary exhibition halls, emergency shelters, earthquake resistant structures, etc.

By **limiting customisation, minimising on scaffolding and layers of construction**, the proposed system is a value addition to the construction industry like India, where 100% customisation is not yet affordable. Customisation is reduced by 45-55% for symmetrical funicular forms and up to 68% for barrel vaults. The construction system also ensures minimising on scaffolding which results in saving cost and time to build curved forms. This system also helps avoiding layers of construction as compared with the Catalan tile vaulting construction. Here the construction is completed in single layer without requirement of skilled labour which in turn saves the cost of craftsmanship and time.

7. Way forward

7.1. CHOICE OF MATERIAL

Three materials were chosen to prototype the modules. M20 concrete, burnt clay and fly-ash. Two primary reasons to choose these materials were, local availability and conventional practices. Other factors considered were, compressive strength of the module and production cost. Based on the analysis done in scan and solve plugin, the maximum compressive strength required is less than 1 MPa. Compressive strength of M20 Concrete is 20 Mpa, burnt clay bricks is 7.35 Mpa and fly-ash bricks is 7.5 Mpa. Hence used for prototyping the module. Although materials like sandstone and limestone has high compressive strength, the cost required for CNC milling them is much higher than the chosen materials.

7.1.1. M20 concrete block

All four types of the blocks, standard, angle spacer, base and key stone were prototyped. Fibre reinforce plastic (FRP) is chosen as material to make moulds of each of them. Scale 1:1 3D printed blocks (Fig 13a) were used to make FRP mould (Fig 13b). A standard M20 mix was made in the laboratory (Fig 13c) and cast in the moulds (Fig 13d). The block was removed from the block after 24 hours and cured for 21 days. Fig 13e to 13h shows final blocks. Structural test, handling and prototyping full scale structure is an ongoing research.

Figure 13. Making of M20 concrete block.

7.2. ASYMMETRICAL VAULTS

An inquiry, if this construction system can be used to build free form funicular vaults?

Acknowledgement

Ketan Solanki, Tanvir Khorajiya, Nikunj Dave and Sameera Bommisetty

References

"OsteoMorphing" : 2019. Available from <http://www.architetturadipietra.it/wp/?p=7080> (accessed 15th March 2019).

Belov, K., Dyskin, A. and Eslein, Y.: 2009, Interlocking of convex polyhedra: Towards a geometric theory of fragmented solids, *Moscow Mathematical Journal*.

Block, P.: 2009, *Thrust Network Analysis: Exploring Three-dimensional Equilibrium*, Ph.D. Thesis, Massachusetts Institute of Technology.

Dallinger, S. and Kolinger, J.: 2008, *Thin post tensioned concrete shell structures, Tailor Made Concrete Structures*, Walraven & Stoelhorst (eds)Taylor & Francis Group, London.

Dyskin, A., Estrin, Y., Pasternak, E., Khor, H.C. and Kanel-Belov, A.: 2003, *Fracture Resistant Structures Based on Topological Interlocking with Non-planar Contacts*, Advanced Engineering Materials.

Rippman, M.: 2016, *Funicular Shell Design: Geometric approaches to form finding and fabrication of discrete funicular structures*, Ph.D. Thesis, University of Stuttgart.

Rippmann, M. and Block, P.: 2013, Funicular Sheel Design Exploration, *Proceedings of ACADIA 2013*.

Tessmann, O.: 2012, Topological interlocking assemblies, *Proceedings of eCAADe 2012*, 211-219.

Vella, I. M. and Kotnik, T.: 2016, Geometric Versatility of Abeille Vault - A Stereotomic Topological Interlocking Assembly. Complexity & Simplicity, *Proceedings of the 34th eCAADe - Volume 2*, Finland, 391-397.

Weizmann, M., Amir, O. and Grobman, J.: 2017, Topological interlocking in architecture: A new design method and computational tool for designing building floors, *International journal of architectural computing*.